

The following report was written by Adam Georgas for the 8th grade Science Fair using the Interactive Instruments Jet Stream 500 Desk top wind tunnel.

Purpose

The purpose of this experiment is to quantitatively determine the structural strength of 3 - dimensional geometric models (an icosahedron, a Fibonacci Golden Ratio model, and a cylinder) when exposed to wind forces in order to provide structurally safe buildings during wind disasters.

Hypothesis

The hypothesis of this experiment is that an icosahedron will withstand greater wind forces than either a Fibonacci figure or a cylinder, of the same volume, because the icosahedron is structurally closer to a sphere and made of 20 connecting triangles, which distributes weight evenly over its entire surface

Procedure

Research: Locate books from the library (I used University of South Alabama Library) that show how to make polyhedrons from paper, descriptions of Fibonacci's Golden Ratio, and Internet research on wind engineering for this experiment.

Experiment:

Making the polyhedrons:

1. Use 200 g/m² (110 lb.) paper to make the polyhedron figures.
2. Make twelve (12) icosahedrons. Draw the pattern shown below to make the icosahedron. The pattern can be drawn using a triangle drawing template or a computer graphics program, like Microsoft Publisher. Triangle sides can be no longer than 4.7 cm to fit on 8 ½" x 11" paper. Fig. 1 shows the pattern using Microsoft Publisher.
3. Cut icosahedron patterns. Refer to icosahedron pattern below and Figure 2 for details to cut the pattern (note flaps along certain lines of the pattern).
4. Brush *Elmer's Glue* on a flap. Tuck the flap beneath (or on top) adjacent triangle in the pattern. As each flap is glued, the three dimensional figure takes shape. Repeat Step 4 until all flaps are glued in place. Consistent technique is important to limit construction differences between the figures. One (1) ml. of glue was used on each model.
5. Constant volume of the figures is the control in the experiment. Determine the volume of the icosahedron using the following equation, $V = (5/12) \times a^3 \times (3 + \sqrt{5})$, (a = length of the edge of the icosahedron). This equation was located at the website www.sisweb.com/math/tables.htm.
6. Knowing the average volume of the icosahedrons:
 - use the equation $V = h \times l \times w$, to find the volume of the Fibonacci model needed.
 - use the equation $V = \pi r^2 h$, where r = radius and h = height of cylinder). Solve for r.
 - the same height was used for the cylinder and Fibonacci model.
7. Draw the patterns for the Fibonacci model and cylinder model of the needed volume. The patterns can also be drawn using Microsoft Publisher or a drawing template.
8. Make 12 Fibonacci models and 12 cylinders using the same materials as listed above. The Fibonacci model uses 0.75 ml. of glue. The cylinder and icosahedron uses 1.0 ml. of glue.
9. An icosahedron and a cylinder will take about 1 hour to make. A Fibonacci model will take 0.5 hour to make.

10. The only uncontrollable variable will be inconsistent construction of the models.
11. Allow the figures to dry for 24 hours.

Conducting the Experiment:

12. Last year, I was not able to get a wind tunnel for the experiment, but held the polyhedrons outside of a moving car to simulate sustained winds. This year, Interactive Instruments, Inc. allowed me to use a small wind tunnel for this experiment. The JetStream 500™ wind tunnel provided more accurate and quantitative measures of the structural strength of each model.
 13. Hot glue the polyhedrons to a 2" x 2" x 1/8" balsa wood base, as in Fig. 3.
 14. Glue a small mirror on to a vertical side of each model. Let dry for 1 hour.
 15. With adult supervision connect the wind tunnel as shown in Figure 4. The diagram shows the system used to measure the drag force and deflection of each model tested. The Jetstream 500™ wind tunnel is described in detail in Appendix A. The wind tunnel is controlled by a personal computer. The JetStream 500 software will measure and record, on the PC, the drag on each model tested.
 16. Use rubber bands to hold the balsa wood base of polyhedron onto the road bed of the wind tunnel, as seen in Figure 5. Aim the mirror directly into the wind hitting the model.
 17. Aim the laser onto the mirror mounted on the model to be tested. The reflected laser beam should be focused on a metric ruler so that reflected beam can be observed when the model deflects. This will give a very measurable and comparable deflection for each model.
 18. Begin testing the polyhedrons at 10 km/h. Observe if any structural deflection or deformation occurs at that speed. If the laser beam moves along the ruler, deflection has occurred. Increase the wind speed in 10 km/h steps, observing the degree of deflection each time. Record relative deflection at each speed.
 19. Repeat step no. 5 through 7 for all polyhedrons tested.
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Materials

The materials needed for this experiment were:

- 200 g/m², 110 lb. paper
 - Microsoft Publisher '97 and WordPerfect 8.0
 - HP DeskJet 660 Cse printer
 - Jet Stream 500™ Wind Tunnel
 - Personal Computer, 450 MHz Pentium II
 - Jet Stream 500 Software
 - Laser pointer
 - Elmer's Glue
 - Nylon paint brush
 - Mirror
 - Glass Cutter
 - Scissors
 - Triple beam balance
 - Metric ruler
 - Hot glue and glue gun
 - 1/8" balsa wood sheets
 - Rubber bands
 - Ear Plugs or muffs
 - 10 ml. Syringe
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Results

The calculated volume of each figure was 245.285cc. The average icosahedron weight was 5.30g, Fibonacci Figure weight was 5.41g, and cylinder average weight was 5.57g. The results of the experiment are shown in the following table.

Wind speed, km/h, at that deflection occurred in each model.			
Figure No.	Icosahedron	Fibonacci Figure	Cylinder
1	40	30	70
2	60	20	80
3	60	30	50
4	80+	20	60
5	60	20	70
6	40	20	70
7	60	40	70
8	80	30	80+
9	60	30	80+
10	80	20	80
11	80	30	80+
12	60	20	80+
Avg.	63.33	42.50	72.50

The average wind speed at which deflection occurred in the icosahedron was 63.33 km/h. Average wind speed of deflection for the cylinder was 72.50 km/h. And the Fibonacci figure suffered deflection at the lowest average wind speed of 42.50 km/h. Using the laser beam to determine the degree of deflection of each model worked very well. The laser beam moved 0.1 to 1.5 cm along the ruler, proportionately with how much the model deflected. This made it easy to compare the relative deflection of each model. model surface facing the wind.

Vibration of each model in the wind stream was also observed. The Fibonacci figure began to vibrate at 30 km/h and increased with wind speed. Vibration was visible in the cylinder at 20-30 km/h. Vibration of the of the Fibonacci figure increased significantly increased with wind speed. The icosahedron had only slight vibration in 40-50 km/h winds and far less than the other models at 80 km/h.

Drag was also measured. Drag is the wind resistance of the model in the wind stream. Reported is the drag measured at the maximum wind speed of the experiment of 80 km/h. The icosahedron recorded the lost average drag of 0.3509 kg, at 80 km/h. Drag on the cylinder was 0.3761 kg and 0.4918 kg for the Fibonacci figure.

Abstract

3-D Geometry Structural Strength Under Wind Forces

Purpose: The purpose of this experiment was to determine the structural strength of 3-dimensional, geometric models (icosahedron, Fibonacci Golden Ratio model, and cylinder) in wind forces.

Hypothesis: The hypothesis was that an icosahedron will withstand greater wind forces than a Fibonacci figure or cylinder because the icosahedron is structurally closer to a sphere, which even distributes weight evenly over its surface

Procedure: Twelve icosahedrons, Fibonacci models, and cylinders were made of 110lb. paper and Elmer's Glue. An Interactive Instruments JetStream500™ wind tunnel was used for this experiment. Models were placed in the wind tunnel. Wind speed was controlled from 0 to 80 km/h, increasing in 10 km/h steps. Drag was measured on a PC. Deflection of the models was measured by reflecting a laser beam onto a mirror glued on the front face of each model and observing the beam's movement. Vibration was also observed.

Results: Results are shown on the charts. On average 42.50 km/h winds caused the Fibonacci figures to deflect. The icosahedrons deflected at 63.33 km/h and the cylinder at 72.50 km/h. The Fibonacci figures and cylinders began to vibrate at 30 km/h and increased with speed. The icosahedron displayed only slight vibration.

Conclusion: Even though the cylinder withstood 8.17 km/h more wind speed, the icosahedron suffered only minor deflection and offered the least drag, much less structural vibration, and the least building materials. For these reasons the icosahedron is the preferable model for buildings exposed to high winds.

Conclusion

The results proved that the hypothesis was partially correct. The icosahedron withstood 21 km/h greater wind speed than the Fibonacci figure and experienced far less structural vibration than the Fibonacci figure. The cylinder, did withstand 8.17 km/h more wind speed than the icosahedron at the point that deflection started.

Even though the cylinder withstood 8.17 km/h more wind speed than the icosahedron, the icosahedron suffered only minor deflection at the highest wind speeds. Also, 9 of 12 icosahedron's offered the less wind resistance (drag) at the highest wind speed. The icosahedron also, experienced much less structural vibration at comparable wind speeds than the cylinder did. Finally, the icosahedron required the least amount (weight) of building materials to enclose the control volume. For this combination of reasons the icosahedron is the preferable model for buildings exposed to high winds.

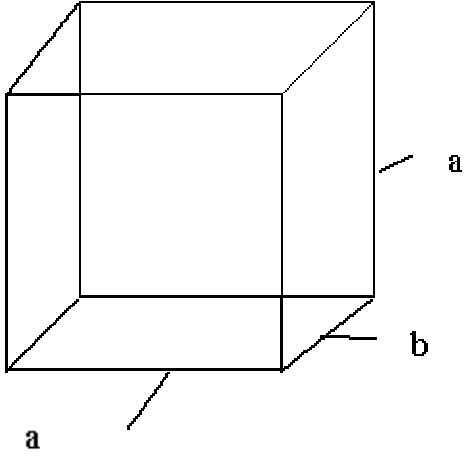
Equations

The volume of the icosahedrons was calculated using the equation,

$$V = (5/12) \times a^3 \times (3 + \sqrt{5}), \text{ where } a = \text{length of edge of the icosahedron.}$$

$$\text{The edge} = 4.73 \text{ cm. } V = (5/12) \times (4.73\text{cm})^3 \times (3 + \sqrt{5}) = 245.285\text{cc.}$$

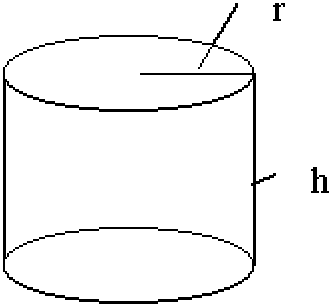
Knowing the volume and that the Fibonacci Ratio is 1.618, the following algebraic formula was used to calculate the dimensions of the Fibonacci figure:

$245.285\text{cc} = a \times a \times b$ $245.285 = a^2 \times 1.618a$ $245.285 = a^3 \times 1.618$ $a = 151.598^{1/3}$ $a = 5.33 \text{ cm}$	<p>Fibonacci Ratio $b/a = 1.618$</p> $b = 1.618 \times a$ $b = 1.618 \times 5.33$ $b = 8.623 \text{ cm}$	
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To test that a figure is a Fibonacci ratio, divide the length by the width, and it should equal 1.618 (or 0.618), such as $8.623/5.33 = 1.61782$.

The dimensions of the cylinder were calculated with the equation:

$V = \pi r^2 h$, where r = radius and h = height of cylinder. The heights of the cylinder and Fibonacci figure were the same, so $h = 8.623$ cm. Solve for the radius of the cylinder as follows:

$V = \pi r^2 8.623 \text{ cm}$ $r^2 = \pi \times 8.623 = 9.05448$ $r = \sqrt{9.05448} = 3.00907 = 3.009 \text{ cm}$	
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Surface area of each model was also calculated with the following:

<u>Icosahedron</u>	<u>Cylinder</u>	<u>Fibonacci figure</u>
$S = 5 a^2 \sqrt{3}$	$S = 2 \pi r h + 2 \pi r^2$	$= 4 (a b) + 2 a^2$
$S = 5(4.73)^2 \sqrt{3}$	$S = 2 \pi (3.00) (8.623) + 2 \pi 3.00907^2$	$= 4 (5.33)(8.623) + 2 (5.33)^2$
$S = 193.76 \text{ cm}$	$S = 219.05 \text{ cm}$	$= 183.84 + 56.82$
		$= 240.66 \text{ cm}$

What is a Cylinder?

A cylinder is a three-dimensional geometric figure. A *circular cylinder* consists of two circular bases of equal area that are in parallel planes, and are connected by a *lateral surface* that intersects the boundaries of the bases. The volume of a circular cylinder is $\pi r^2 h$, where r is the radius of the bases, and h is the perpendicular distance between the planes that contain the bases. In a *right circular cylinder*, the lateral surface is perpendicular to the bases. The lateral surface area of a right circular cylinder is $2\pi rh$, and the total surface area is $2\pi r(r+h)$. Cylindrical structures are seen in Marina Towers (Chicago), oil storage tanks, and cooling towers for power plants.

WHAT IS A FIBONACCI?

Leonardo Fibonacci was an Italian mathematician who lived during the 12th-13 century. Hew collected and added to mathematics in the fields of algebra and number theory. He is most famous for his discovery, the Fibonacci Number Series.

FIBONACCI NUMBER SERIES

0+1 1+1 1+2 2+3 3+5 5+8 8+13 13+21 21+34 34+55 55+89
1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144,

FIBONACCI RATIOS

A Fibonacci ratio consists of any Fibonacci number divided by another Fibonacci number. The ratio usually consists of adjacent numbers in the series. The lager number can be divided by the smaller number, which results in 1.61. If the smaller number is divided by the lager number, the answer is the reciprocal, 0.61.

FIBONACCI'S GOLDEN RECTANGLE

The Fibonacci rectangle is known as the "Golden Rectangle" because people unconsciously favor it. It strikes people as quite "perfect," not too fat and stubby nor too long and skinny. The Golden Rectangle results when the Fibonacci ratio is used and the sides are 1:1.61803. Interestingly, if a square is cut from the Golden Rectangle, the remaining rectangle will also be a Golden Rectangle. Golden Rectangles are seen everywhere: 3 x 5 and 5 x 8 index cards, credit cards, the Parthenon of ancient Greece, ranch style houses, trailers, and conversion/mini vans.

WHAT IS AN ICOSAHEDRON ?

An icosahedron is a polyhedron or geometric figure made up of twenty triangular sides. The icosahedron most closely resembles a geodesic dome with its repeated triangular faces. The triangle is basic for all forms of structural stability since it is the only construction that is truly rigid. Each triangle in the icosahedron shifts whatever stress is applied to it equilaterally throughout the other triangles. Interestingly, a geodesic structure was the only building to withstand the atomic bomb blast at Hiroshima. Where do we see geodesic structures? We see domes on Orthodox churches, the dome at Epcot Center in Disney World, the Montreal Expo Dome, the Climatron (a gigantic climate controlled botanical garden in St. Louis), and the Houston Astrodome.

Deflection Calculation Spreadsheets

Icosahedron Deflection vs Wind Speed								
	Wind Speed, kph							
Model No.	10	20	30	40	50	60	70	80
1	0.0	0.0	0.0	0.1	0.1	0.1	0.3	0.6
2	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.2
3	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.2
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1
6	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.2
7	0.0	0.0	0.0	0.0	0.0	0.1	0.2	0.2
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1
9	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.2
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1
11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1
12	0.0	0.0	0.0	0.0	0.0	0.1	0.2	0.2
Avg	0.00	0.00	0.00	0.01	0.02	0.06	0.10	0.18
Std Dev	0	0	0	0.028868	0.038925	0.051493	0.095346	0.14668
Fibonacci Figure Delfelction vs Wind Speed								
	Wind Speed, kph							
Model No.	10	20	30	40	50	60	70	80
1	0	0	0.1	0.3	0.4	0.5	0.5	0.6
2	0	0.1	0.2	0.1	0.1	0.1	0.1	0.2
3	0	0	0.1	0.1	0.2	0.3	0.4	0.4
4	0	0.1	0.2	0.2	0.2	0.3	0.4	0.4
5	0	0.1	0.1	0.2	0.2	0.2	0.2	0.3

6	0	0.2	0.4	0.6	0.6	0.8	0.9	1.1
7	0	0.1	0.1	0.1	0.1	0.2	0.2	0.3
8	0	0	0.1	0.1	0.1	0.1	0.1	0.1
9	0	0	0.1	0.2	0.3	0.5	0.6	0.7
10	0	0.1	0.2	0.4	0.5	0.7	1	1.1
11	0	0	0.2	0.4	0.9	1.1	1.3	1.5
12	0	0.1	0.1	0.5	0.5	0.5	0.6	0.7
Average	0.00	0.07	0.16	0.27	0.34	0.44	0.53	0.62
Std Dev	0	0.062361	0.086201	0.164992	0.236144	0.292855	0.363146	0.407908
Cylinder Deflection vs Wind Speed								
	Wind Speed, kph							
Model No.	10	20	30	40	50	60	70	80
1	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.2
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1
3	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1
4	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.2
5	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1
6	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.2
7	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.2
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2
11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Average	0.00	0.00	0.00	0.00	0.01	0.02	0.05	0.11
Std Dev	0	0	0	0	0.028868	0.038925	0.052223	0.090034